Likelihood Demonstration

Jim Chiang
GSSC/UMBC/SLAC

Likelihood Tutorials

- **Unbinned**

  http://glast-ground.slac.stanford.edu/workbook/science-tools/pages/sciTools_likelihoodTutorial/likelihoodTutorial.htm

- **Binned**

  http://glast-ground.slac.stanford.edu/workbook/science-tools/pages/sciTools_binnedLikelihoodTutorial/binnedLikelihood.htm

- **Running from Python**

  http://glast-ground.slac.stanford.edu/workbook/science-tools/pages/sciTools_pyLikelihood_tutorial/pythonTutorial.htm

- **Workbook Home Page**

  http://glast-ground.slac.stanford.edu/workbook/

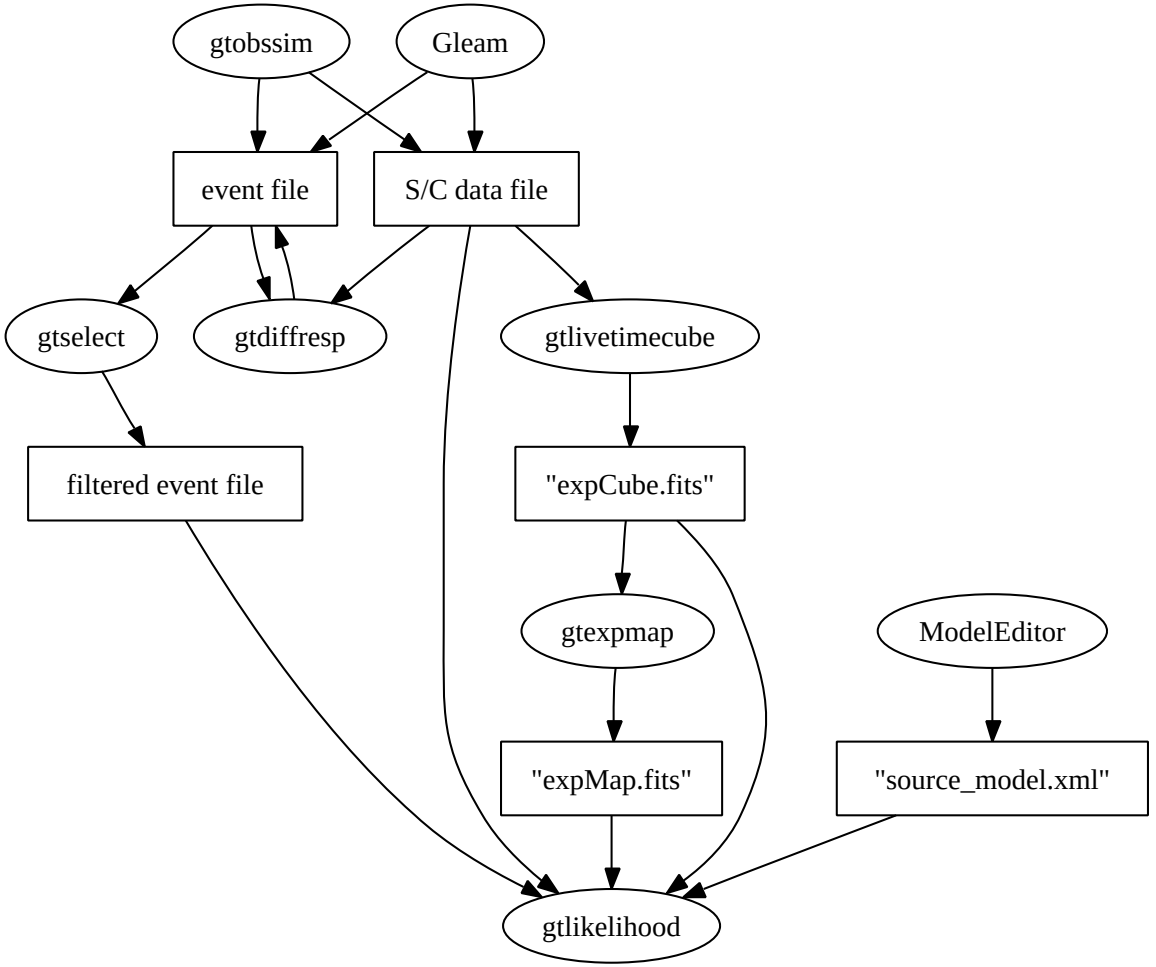
Binned Analysis Demo

- **gtcntsmap** → **countsMap.fits**: for display, summed over all energies
- **ds9 countsMap.fits**: show data to ID sources
- **ModelEditor** → **anticenter_model.xml**: extract 3EG sources
- **gtcntsmap** → **countsMap_20.fits**: for analysis with 20 energy bins
- gtlivetimecube → expCube.fits: live-time calculation as a function of sky position and off-axis angle
- **gtsrcmaps** → **srcMaps.fits**: multiply source model by exposure, convolve with energy-dependent PSF.
- **gtlikleihood** → **anticenter_fit.xml**: binned analysis
- **gtmodelmap** → **model_map.fits**: create model counts map
- **ds9 model_map.fits**: compare/blink with countsMap.fits

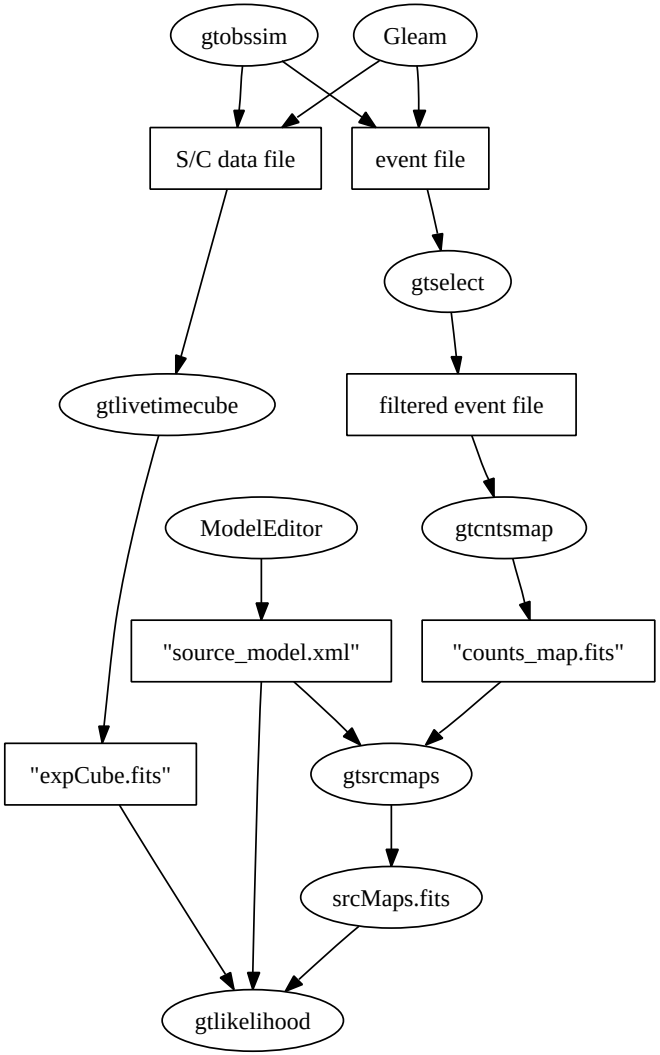
Unbinned Analysis Demo from Python

- gtselect → filtered.fits: define region-of-interest
- gtdiffresp → filtered.fits: add event-specific diffuse model response information
- gtexpmap → expMap.fits: exposure calculation for unbinned analysis
- **ipython**: Run an interactive analysis from Python
  - **run analysis**: startup script using binned fit
  - **print foo.model**: show model status
  - **ModelEditor()**: edit anticenter_model.xml to use broken power-law for Crab and Geminga
  - **bar = UnbinnedAnalysis(obs, 'anticenter_bpl_model.xml')**
  - **bar.plot()**
  - **bar.fit()**
  - **bar.oplot()**

Unbinned Likelihood Overview



Binned Likelihood Overview



FTOOLs used in the Likelihood analysis

- `gtselect`: Select events based on time, sky location, energy, conversion layer. These selections define the region-of-interest for unbinned analysis and so are required in that case. Time selections can be made with this tool for use in binned analysis for variability studies.
- `gtdiffresp`: Pre-compute exposure quantities related to diffuse sources of the sky model such as interstellar emission and an isotropic extragalactic component. These quantities are added as columns to the event files. It is often a time-saver to run this on a large dataset first, then make sub-selections using `gtselect`.
- `gtlivetimecube`: Integrate livetime as a function of sky location (RA, Dec) and off-axis angle. These livetime integrals are used to accelerate the exposure map calculations.
- `gtexpmap`: Performs an exposure calculation for use by unbinned analysis only. The resulting map is needed to analyze diffuse sources.

- `gtcntsmap`: Create a counts map with image planes logarithmically spaced in energy. This is used only for the binned analysis.
- `gtsrcmaps`: Convolve source model components with the energy dependent PSF. The geometry of the maps created for each source match the counts map which is to be analyzed by `gtlikelihood`.
- `ModelEditor`: A prototype GUI to edit the XML source model files. Some functionality is still missing, e.g., the ability to specify template FITS files for diffuse sources, so some hand editing using a text editor may be needed. A more detailed description is here
  http://confluence.slac.stanford.edu/display/ST/Source+Model+Editor
- `gtlikelihood`: The FTOOL-like likelihood application.

Statistical Models

- Instrument response

$$R(E', \hat{p}'; E, \hat{p}, t) = A(E, \hat{p}, t) P(\hat{p}'; E, \hat{p}, t) D(E'; E, \hat{p}, t). \quad (1)$$

  $E, \hat{p}$ = true photon energy and momentum; $E', \hat{p}'$ = measured energy and momentum.

- Unbinned likelihood

$$\log \mathcal{L} = \sum_j \log M(E'_j, \hat{p}'_j, t_j) - N_{\rm pred} \quad (2)$$

  where

$$M(E', \hat{p}', t) = \sum_i \left[\int_{\rm SR} dE d\hat{p}\, R(E', \hat{p}', t; E, \hat{p})\, S_i(E, \hat{p}') \right], \quad (3)$$

$$N_{\rm pred} = \int_{\rm ROI} dE' d\hat{p}' dt\, M(E', \hat{p}', t) \quad (4)$$

  Sources are indexed by i; events are indexed by j. SR is the "source region"; ROI is the "region-of-interest".

- Binned likelihood

$$\mathcal{L} = \prod_j \frac{\theta_j^{n_j} e^{-\theta_j}}{n_j!} \tag{5}$$

where

$$\theta_{ij} = \int_j dE'\, d\hat{p}' \int dt \int_{\rm SR} dE\, d\hat{p}\, R(E', \hat{p}'; E, \hat{p}, t) S_i(E, \hat{p}), \tag{6}$$

$$\theta_j = \sum_i \theta_{ij}. \tag{7}$$

n_j is the number of events in pixel j.

- Current implementation neglects energy dispersion, i.e.,

$$D(E'; E, \hat{p}, t) \equiv \delta(E - E') \tag{8}$$